

Chapter 2

Reasoning and Proofs



2.1 - Conditional Statements

2.2 - Inductive and Deductive Reasoning

2.3 - Postulates and Diagrams

2.4 - Algebraic Reasoning

2.5 - Proving Statements about Segments and Angles

2.6 - Proving Geometric Relationships

2.1 - Conditional Statements

*If a snake has a red, yellow, and black pattern, **then** it is a King Snake.*



Conditional statement - a logical statement that has two parts, a *hypothesis* (p) and a *conclusion* (q).

If p , then q . or $p \longrightarrow q$

For example: If an animal is a bird, then it has feathers.

hypothesis

conclusion

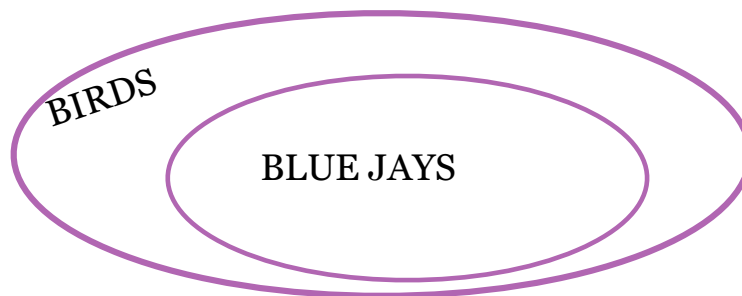
2.1 - Conditional Statements

Examples:

- *If today is Thanksgiving Day, then today is Thursday.*
- *A number is a rational number if it is an integer.*

Write a conditional:

- *An obtuse triangle has exactly one obtuse angle.*
- Write a conditional for the Venn diagram:



2.1 - Conditional Statements

A **conditional statement** has a truth value of either true (T) or false (F).

For example:

- If the animal is a bluejay, then it is a bird.
- If today is Friday, then I win the lottery.
- If I own a spaceship, then the sky is red.

Hypothesis p	Conclusion q	Conditional p -> q
T	T	T
F	T	T
T	F	F
F	F	T

2.1 - Conditional Statements

Vocabulary

Negation - the opposite of the original statement.

p becomes $\sim p$ or “not p ”

an animal is a bird

p

an animal is not a bird

$\sim p$

2.1 - Conditional Statements

- **Conditional:** If p then q .
- *If two angles are congruent, then they are acute.*
- **Converse:** exchanging the hypothesis and conclusion.

If q then p or $q \longrightarrow p$

- *If two angles are acute, then they are congruent.*

2.1 - Conditional Statements

- **Conditional:** If p then q .
- *If two angles are congruent, then they are acute.*
- **Inverse:** negate the hypothesis and conclusion.

If $\sim p$ then $\sim q$ or $\sim p \longrightarrow \sim q$

- *If two angles are not congruent, then they are not acute.*

2.1 - Conditional Statements

- **Conditional:** If p then q .
- *If two angles are congruent, then they are acute.*
- **Contrapositive:** negate the hypothesis and conclusion, then exchange them.

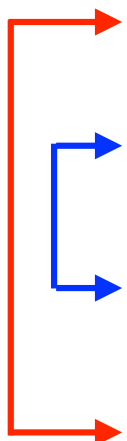
If $\sim q$ then $\sim p$ or $\sim q \longrightarrow \sim p$

- *If two angles are not acute, then they are not congruent.*

2.1 - Conditional Statements

- **Logically equivalent statements:** conditional statements that have the same truth value.

Statement	Example	Truth Value
Conditional	<i>If two angles are both 45°, then they are congruent.</i>	<i>T</i>
Converse	<i>If two angles are congruent, then they are both 45°.</i>	<i>F</i>
Inverse	<i>If two angles are not both 45°, then they are not congruent.</i>	<i>F</i>
Contra-positive	<i>If two angles are not congruent, then they are not both 45°.</i>	<i>T</i>



Equivalent

2.1 - Conditional Statements

Example:

Let p be “you are a guitar player” and let q be “you are a musician.” Write the following:

1. the conditional statement
2. the converse
3. the inverse
4. the contrapositive

2.1 - Conditional Statements

- **Biconditional Statement:** a statement that can be written in the form “***p if and only if q.***”
- This form means **both** the **conditional** statement and its **converse**.
“*if **p** then **q***” and “*if **q** then **p***”

- Example: “An angle is obtuse ***if and only if*** its measure is greater than 90 degrees and less than 180 degrees.”
- Conditional:
- Converse:

2.1 - Conditional Statements

- **Biconditional Statement:** a statement that can be written in the form “*p if and only if q.*”
- This form means **both** the **conditional** statement and its **converse**.
“*if p then q*” and “*if q then p*”

- Example: “A solution is neutral \longleftrightarrow its pH is 7.”
- Conditional:
- Converse: